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Introduction to "Flux-Corrected Transport. I. SHASTA, A Fluid Transport Algorithm That Works"

In 1973, in the eighth year of its youth, the *Journal of Computational Physics* published the classic Boris and Book paper describing flux-corrected transport (FCT) [1]. Almost all of the monotonicity-preserving and nonoscillatory fluid transport algorithms of today trace their origins, ultimately, to ideas that first appeared in this paper.

Boris and Book's new and far-reaching idea was to locally replace formal truncation error considerations with conservative monotonicity enforcement in those places in the flow where the formal truncation error had lost its meaning, i.e., where the solution was not smooth and where formally high order methods would violate physically motivated upper and lower bounds on the solution. This is today still the fundamental principle underlying the great bulk of the monotonicity-preserving and nonoscillatory algorithms that have appeared in more recent times. Occasionally this bit of history is lost in some of the more recent literature, in part due to the fact that the paper is now 24 years old (and the original publication [2] older still).

In [1], the authors applied this fundamental idea to a specific algorithm they termed SHASTA. They were able to show not only sharp monotone advection of linear discontinuities, but also sharp nonoscillatory gasdynamic shock waves. Included in [1] was a SHASTA calculation of a shock tube problem much more difficult than that used by Sod five years later [3], with nearly monotone results and with no knowledge of the solution (e.g., Riemann solvers) built in to the algorithm. All of these calculations were the first of their kind with monotonicity-preserving algorithms of greater than first-order accuracy. It was also in this paper that the term "flux-limiting" [1, 50] appeared in print for the first time.

In the years following 1973, Boris and Book and colleagues published two more FCT papers in the *Journal of Computational Physics* [4, 5], followed by a chapter in the *Methods in Computational Physics* book series [6] that summarized their work through 1976. These works refined their ideas, generalizing the algorithms to a larger class of which SHASTA was just one member. Their emphasis was on the continuity equation as a scalar representative of systems of conservations laws and on advective phase error as a primary culprit in the elimination of the errors that remained after nonoscillatory behavior was eliminated via flux limiting. A more recent summary of FCT is given in the book by Oran and Boris [7].

Work on FCT algorithms has also thrived elsewhere in publications far too numerous to reference here. Two notable examples are the extension of FCT to fully multidimensional form in 1979 [8] and the generalization of FCT to unstructured grids (e.g., triangles and tetrahedra in two and three dimensions, respectively) by Parrott and Christie in 1986 [9]. One of the consequences of this last development has been the ability to perform FCT calculations in extremely complex geometries. An example is the remarkable simulation of the World Trade Center blast which modeled in detail the garage of the building including all of the parked cars [10].

The response of the scientific computing community to FCT was and still is remarkably strong. The original paper alone [1] has been cited 513 times, according to the ISI database. Even more telling is that 238 of these citations were during the 1990s. This is an astounding total, given that these citations were all for a paper that was at least 17 years old at the time of the citation! Clearly the impact of this paper is still being felt long after its original publication. FCT has been applied to virtually every area of science, from aerodynamics and shock physics to atmospheric and ocean constituent transport, magnetohydrodynamics, kinetic and fluid plasma physics, astrophysics, and computational biology.

Before releasing the reader to enjoy the paper, allow me to give a personal view of the relationship between FCT and the "nonoscillatory upwind schemes" that appeared later, started by the pioneering work of Van Leer [11, 12]. If one examines any of these upwind schemes he will find that, in addition to the machinery which makes them upwind, they inevitably contain monotonicity constraints or "limiters" whose function is identical to that of the Boris–Book flux limiter: to impose monotone nonoscillatory behavior where formally high order methods would do otherwise. Furthermore, the form of these limiters is quite similar to that of the original Boris–Book limiter, but of course, this is to be expected, given their common goal. Van Leer, to whom these constraints are most often attributed within the upwind context, clearly developed his ideas independently (and within a year following those of Boris and Book), but given the earlier publication, it is

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in my view only fair to attribute them to Boris and Book as well.

From this observation arises an obvious question: how important are these 24-year-old monotonicity constraints in the modern ‘‘nonoscillatory upwind schemes’’ of today? Are they really important anymore, or is upwindedness more so? Let me propose the following experiment, which will suggest to the reader that even today they are the critical component of these schemes: Consider advection in one dimension (one dimension being the upwind methods’ arena of greatest strength) and consider the excellent MUSCL method [12] as an example of an upwind method. What would adversely affect the performance of MUSCL more: removal of the monotonicity constraint (slope limiter), or removal of the upwindedness? We could easily remove the upwindedness by replacing the characteristic tracing with a Hancock half-step [13], and using something dissipative but centered for the Riemann solver, e.g. the Rusanov method. What we will find is that the algorithm is somewhat more dissipative than the original, but still quite good for a second-order method, and most important, monotonocity preserving. But if we were to remove the slope limiter, our results would be disastrous no matter how upwind we were, no matter how careful our characteristic tracing, and no matter how good our Riemann solver. One could conduct other similar experiments with other upwind and nonupwind methods, e.g. [14], and would reach similar conclusions. Clearly the monotonicity constraints are critical components of these modern methods. And those constraints are born of ideas that first appeared in the paper that follows.

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Steven T. Zalesak

NASA Goddard Space Flight Center
Greenbelt, Maryland 20771